current on the surface, and the ion accommodation and neutralization processes; $T$, $\bar{T}$, $a$, $\bar{a}$, $\lambda$, $\bar{\lambda}$, temperature, thermal diffusivity, and thermal conductivity in the solid and liquid phases, respectively; $T_m$, characteristic temperature; $\rho$, density; $L$, $L_0$, latent heats of melting and evaporation; $v_0$, maximum velocity of evaporation-front motion; $\bar{x}$, running coordinate; t, time; $\bar{y}_0(t)$, $\bar{y}(t)$, coordinates of the evaporation and melting fronts; $\tau = t/t^\circ$, $\xi = x/l^\circ$, $\theta = T/T^\circ$, $\psi(t) = y(t)/l^\circ$, $u_0 = v_0/v^\circ$, dimensionless time, running coordinate, temperature, melting-front coordinate, and maximum velocity of evaporation front motion; $t^\circ = \bar{a}/(v^\circ)^2$, $l^\circ = v^\circ t^\circ$, $T^\circ$, characteristic time, length, and temperature; $v^\circ$, evaporation-wave velocity in the body:

$$\delta_\xi \theta_{i,n} = \frac{1}{h}(\theta_{i+1,n} - \theta_{i,n}),\quad \delta_{\bar{\xi}\xi}\theta_{i,n} = \frac{1}{h^2}(\theta_{i+1,n} - 2\theta_{i,n} + \theta_{i-1,n}),$$

$$\delta_\tau \theta_{i,n} = \frac{1}{\Delta\tau_n}(\theta_{i,n} - \theta_{i,n-1});$$

$\tilde{\tau}$, duration of the flat top of the current pulse; U, charging voltage on the capacitors; $\bar{x}_a$, coordinate of the deterioration boundary (experiment) on the anode; $\bar{x}_c$, coordinate of the deterioration boundary (experiment) on the cathode.

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# CHOOSING A CALCULATION SCHEME FOR THE TEMPERATURE FIELD IN PULSE MICROWELDING


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Thermal processes in pulsed methods of microwelding in a solid phase during semiconductor assembly are investigated. Expressions determining the choice of the temperature-field calculational scheme are subjected to numerical analysis. It is shown that the calculated and experimental data are in qualitative agreement.


In choosing the optimal technology and equipment for the assembly of microelectronic devices, an important stage is the determination of the acceptable thermal effect on semiconductor elements of the microwelding in the solid phase of a small-diameter wire to contact areas on its surface.


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Investigation of the temperature field for the most promising of the currently available pulse methods of welding (ultrasound welding and welding by a "split" electrode) has assumed [1] that the semiconductor crystal is radially unbounded, that welding occurs under the action of a circular heat source of constant power, that the upper face of the crystal is adiabatic, and that the temperature in the welding cycle reaches its steady value sufficiently rapidly for the unsteady component to be neglected in calculations. For a series of specific cases in practice, these conditions are not satisfied due to the finite dimensions of the crystal, to asymmetry of the heat-liberation zone, which is most often elliptical, and to loss of heat from the boundary surfaces.

The solution of such practical problems involves considerable difficulties, and numerical solution is not usually possible. Therefore, an important stage in the thermophysical analysis of thermal processes in microwelding is the intelligent schematization of the process itself, of the form of the body involved in it, and of the laws describing the distribution of the heat sources. The reliability and practical value of the investigation depends to a considerable extent on the chosen calculation scheme.

The aim of the present paper is to obtain relations giving the limits within which specific calculational schemes can be used and, on this basis, to obtain simplified expressions describing the thermal processes in pulse methods of microwelding on the surfaces of semiconductor crystals. First of all, we obtain approximate expressions for limiting cases with the simplest boundary conditions, the results of which may be applied to more complex calculational schemes, and then we consider a semiinfinite body with a local heat source of constant power acting at its surface.

The condition for which a body may be regarded as semiinfinite may be obtained by means of the expression

$$\frac{T(0,\ h_\infty)}{T(0,\ 0)} = \frac{\sqrt{h_\infty^2 + r_0^2} - h_\infty}{r_0} \leqslant \delta, \tag{1}$$

where $\delta \leq 0.05$; $T(0,\ 0)$ is the value of the temperature at the center of a circular heat source of constant power on the surface of a semibounded body; $T(0,\ h_\infty)$ is the temperature at a distance $h_\infty$ (in the direction of the z axis) from the center of the source [2]. Hence

$$h_\infty \geqslant \frac{1-\delta^2}{2\delta} r_0 = 9.975 r_0. \tag{2}$$

If $h \geq h_\infty$, the condition of heat exchange on the opposite side of the crystal introduces no marked distortion of the temperature field throughout the volume of the semiconductor device. If $h < h_\infty$, it is necessary to solve a two-layer problem, similar to that in [1]. The condition at which the elliptical heat source may be taken as equivalent to a circle is obtained as follows.

A continuously acting point source of power $Q_0$ situated at the point $(x_0,\ y_0,\ 0)$ on the surface of a semibounded space [3] produces a temperature

$$T(x,\ y,\ z,\ x_0,\ y_0,\ 0) = \frac{Q_0}{2\pi\lambda \sqrt{(x-x_0)^2 + (y-y_0)^2 + z^2}}. \tag{3}$$

The temperature due to an elliptical heat source may be obtained according to the superposition principle, by integrating Eq. (3) over an ellipse with semiaxes $a$ and b and center at the origin of the coordinates. After integrating, we obtain

$$T(x,\ y,\ z) = \frac{q_0}{2\pi\lambda} \iint\limits_{\left(\frac{x_0^2}{a^2} + \frac{y_0^2}{b^2} = 1\right)} \frac{dS}{\sqrt{(x-x_0)^2 + (y-y_0)^2 + z^2}} = \frac{2q_0 ab}{\lambda} \int_0^a \ln u\, dx_0,$$

$$u = \left[\frac{y + \sqrt{(x-x_0)^2 + y^2 + z^2}}{y - b\sqrt{1 - \frac{x_0^2}{a^2}} + \sqrt{(x-x_0)^2 + \left(y - b\sqrt{1 - \frac{x_0^2}{a^2}}\right) + z^2}}\right] \tag{4}$$

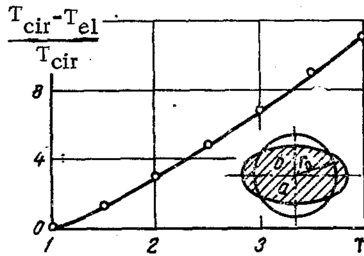


Fig. 1

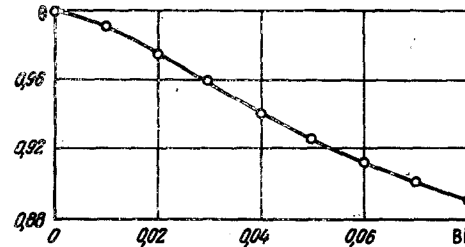


Fig. 2

Fig. 1. Acceptable error, %, in calculating temperature from a circular heat source rather than the equivalent elliptical heat source as a function of the ratio between the ellipse semiaxes.

Fig. 2. Dependence of the dimensionless temperature on the conditions of heat exchange at the crystal surface ($r_0 = 4 \cdot 10^{-5}$ m, $\xi = 0$, $\chi = 0$).

The temperature at the center of a circular heat source is

$$T_{cir}(0, 0, 0) = \frac{q_0 r_0}{\lambda}, \quad (5)$$

where $r_0 = \sqrt{ab}$ is determined from the condition that the circular and elliptical sources are of equal area.

It will be assumed that if

$$\frac{T_{el}(0, 0, 0)}{T_{cir}(0, 0, 0)} = -\frac{2}{\pi\sqrt{\gamma}}\left(1 + \int_0^1 \ln\left(\sqrt{x_0^2 + \gamma^2(1 - x_0^2)} - \gamma\sqrt{1 - x_0^2}\right) dx_0 \geqslant \delta, \quad (6)$$

where $\delta \geq 0.95$, $\gamma = b/a$, the asymmetry of the heat-liberation zone will not lead to significant distortion of the heat field during welding and the elliptical source may be replaced by an equivalent circular source. Numerical solution of Eq. (6) (Fig. 1) shows that if

$$\gamma \leqslant 2{,}6 \quad (7)$$

a simplified calculation scheme may be used.

The condition which allows the crystal to be considered as radially unbounded is found using the expression for the temperature field from a circular heat source of constant power [2]:

$$T(r, z) = \frac{q_0 r_0}{\lambda} \int_0^\infty J_1(x_0 r_0) J_0(x_0 r) \exp(-x_0 z) \frac{dx_0}{x_0}, \quad (8)$$

where $J_1(x_0 r_0)$, $J_0(x_0 r)$ are Bessel functions of first order and zeroth order.

To reduce Eq. (8) to a simpler form, assume that $J_1(x_0 r_0) J_0(x_0 r)/x_0$ is original and that the corresponding Laplace operator representation is [4]

$$T(r, z) = \frac{q_0}{\pi\lambda} \int_{-r_0}^{r_0} \frac{\sqrt{r_0^2 - x_0^2}\, dx_0}{\sqrt{r^2 + (z + ix_0)^2}}. \quad (9)$$

For $r \leq r_0$, we set $z = 0$ and make the substitution $x_0 = r \sin\varphi$, to obtain

$$T(r, 0) = \frac{2q_0 r_0}{\pi\lambda} \mathrm{E}\left(\frac{r^2}{r_0^2}, \arcsin\frac{r_0}{r}\right), \quad (10)$$

while for $r > r_0$ the substitution $x_0 = r_0 \sin\varphi$ gives

$$T(r, 0) = \frac{2q_0 r}{\pi\lambda}\left[\left(\frac{r_0^2}{r^2} - 1\right)\mathrm{K}\left(\frac{r_0^2}{r^2}\right) + \mathrm{E}\left(\frac{r_0^2}{r^2}\right)\right], \quad (11)$$

where $E[r^2/r_0^2, \arcsin(r_0/r)]$ is a normal second-order elliptical integral; $K(r_0^2/r^2)$ is a complete first-order elliptical integral; $E(r_0^2/r^2)$ is a complete second-order elliptical integral.

For $r > r_0$ we introduce the inequality

$$\frac{T(r_\infty, 0)}{T(0, 0)} = \frac{2r_\infty}{\pi r_0}\left[\left(\frac{r_0^2}{r_\infty^2} - 1\right) K\left(\frac{r_0^2}{r_\infty^2}\right) + E\left(\frac{r_0^2}{r_\infty^2}\right)\right] \leqslant \delta, \tag{12}$$

where $\delta \leq 0.05$ and

$$r_\infty \geqslant 10.00 r_0. \tag{13}$$

When $l \geq r_\infty$, the heat-exchange condition at the crystal face does not affect the temperature of the semiconductor device during microwelding and in this case the crystal may be considered as a radially unbounded body. Numerical solution of Eq. (12) is carried out using a table of elliptical integrals [5].

To take into account the heat loss at the upper face of the crystal and to derive the condition under which it may be neglected, we consider the following boundary conditions at the crystal surface:

$$\lambda \frac{\partial T}{\partial z} + q_0 = 0 \quad \text{for} \quad z = 0;\ 0 \leqslant r \leqslant r_0, \tag{14}$$

$$\lambda \frac{\partial T}{\partial z} - \alpha T = 0 \quad \text{for} \quad z = 0;\ r > r_0. \tag{15}$$

If it is assumed that the heat exchange occurs over the entire upper face of the crystal, the temperature can be given in dimensionless form [2],

$$\theta = \int_0^\infty J_1(x_0) J_0(x_0 \xi) \exp(-x_0 \chi) \frac{dx_0}{x_0 + \mathrm{Bi}}. \tag{16}$$

Converting Eqs. (14) and (15) to dimensionless form gives

$$-\frac{\partial \theta}{\partial \chi} = 1 \quad \text{for} \quad \chi = 0;\ 0 \leqslant \xi \leqslant 1, \tag{17}$$

$$\frac{\partial \theta}{\partial \chi} = \mathrm{Bi}\theta \quad \text{for} \quad \chi = 0;\ \xi > 1. \tag{18}$$

Equation (16) satisfies the boundary conditions only if $\xi > 1$.

Thus Eq. (16) describes the case when, in addition to the constant heat flux under the source directed into the body of the part, there exists a flux varying over the area of the source, directed into the surrounding medium. Equation (16) corresponds to the heat exchange occurring over the whole surface of the crystal, including the zone of action of the heat source.

When Eqs. (17) and (18) are not satisfied, the problem can be solved by the procedure in [6], essentially as follows. If in Eq. (16) the heat loss through the area occupied by the source is determined and compensated by the addition of a heat source of corresponding power, it is possible to obtain a solution integrally satisfying Eq. (17) and locally satisfying Eq. (18). Thus, Eq. (17) can be written in integral form as

$$-\pi r_0^2 \frac{\partial \theta}{\partial \chi} = \pi r_0^2 \quad \text{for} \quad \chi = 0;\ 0 \leqslant \xi \leqslant 1, \tag{19}$$

while Eq. (16) integrally satisfies the condition

$$-\pi r_0^2 \frac{\partial \theta}{\partial \chi} = \pi r_0^2 - 2\pi \mathrm{Bi} \int_0^{r_0} \theta \xi d\xi \quad \text{for} \quad x = 0;\ 0 \leqslant \xi \leqslant 1, \tag{20}$$

which may differ significantly from Eq. (19), depending on the value of Bi. If Eq. (19) is to be satisfied, Eq. (16) must be increased by a factor of n, where n is a number determined from the relation

$$\left( r_0^2 - 2\mathrm{Bi} \int_0^{r_0} \theta \xi d\xi \right) n = r_0^2. \tag{21}$$

Multiplying Eq. (16) by n, as defined in Eq. (21), gives an expression integrally satisfying Eq. (19) under the source and locally satisfying Eq. (18) elsewhere.

This solution is not accurate but, because of its simplicity, it allows a number of satisfactory practical conclusions to be drawn.

A numerical calculation of Eqs. (16) and (21) is shown in Fig. 2. The coefficient of heat exchange between the crystal and the surrounding medium is calculated on the basis of the radiant component only (since the convective component is small), from the formula

$$\alpha = \varepsilon\sigma \frac{T_m^4 - T_0^4}{T_m - T_0}, \tag{22}$$

where $T_m$ is the maximum temperature on the crystal surface.

It will be assumed that if

$$\frac{T\alpha(0, 0)}{T_{\mathrm{cir}}(0, 0)} \geqslant 0.95, \tag{23}$$

where $T_\alpha(0, 0)$ is the temperature at the center of a circular heat source calculated from Eqs. (16) and (21), the upper crystal face is thermally insulated.

Taking into account the specific properties of the microwelding process, the completed analysis of the boundary conditions allows a number of simplifying assumptions to be made.

1. For the majority of commonly used types of IS semiconductor device on a standard base, the crystal thickness lies in the range $(2\text{-}4)\cdot 10^{-4}$ m, while the connection between the crystal and the base is made by small-diameter wire, in most cases $(2.4\text{-}4)\cdot 10^{-5}$ m. Therefore, as estimates show, Eq. (12) is satisfied accurately, or with an error not exceeding 3%; in this case, $h_\infty = 6.21$. Therefore, the crystal will be regarded as unbounded in the direction of the z axis.

2. Since, in microwelding, a ratio of more than 1:25 between the semiaxes of the elliptical welding zone is untypical, and taking account of the asymmetry of the heat source leads to no significant improvement in accuracy, the temperature calculation is carried out for a circular heat source.

3. From our analysis, it is clear that heat exchange with the surrounding medium introduces significant distortion only for very large values of temperature at the crystal surface [from Eq. (22), T >> 1000°K]. Since microwelding in the solid phase proceeds at a temperature below the eutectic temperature of the joined materials (<773°K), the boundary surface of the crystal may be assumed to be adiabatic.

Taking into account that the IS contact area lies on the periphery of the semiconductor crystal, Eq. (12) is not satisfied, as estimates show. Introducing the above simplifications, the temperature field can be found using Eq. (8) in the Cartesian coordinate system X, Y, Z. If the center of the heat source is at the point $(0, -l, 0)$, then

$$T_1(x, y+l, z) = \frac{q_0 r_0}{\lambda} \int_0^\infty J_1(pr_0)\, J_0\left(x_0 \sqrt{x^2 + (y+l)^2}\right) \exp(-x_0 z) \frac{dx_0}{x_0}. \tag{24}$$

Since, as was shown above, the crystal boundary plane is adiabatic, it is necessary to introduce an analogous hypothetical heat source with center at the point $(0, l, 0)$,

$$T_2(x, y-l, z) = \frac{q_0 r_0}{\lambda} \int_0^\infty J_1(pr_0)\, J_0\left(x_0 \sqrt{x^2 + (y-l)^2}\right) \exp(-x_0 z) \frac{dx_0}{x_0}, \tag{25}$$

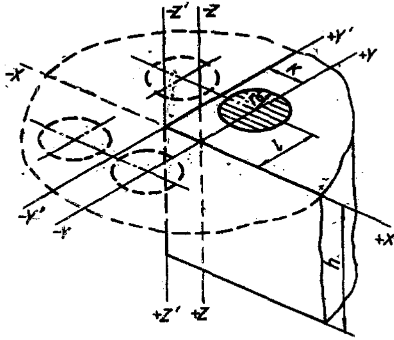


Fig. 3

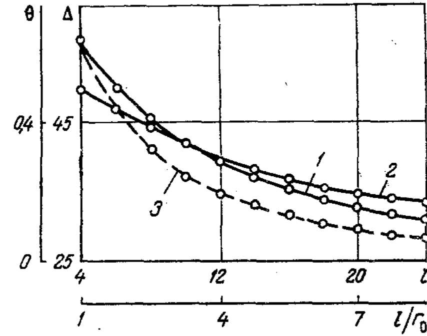


Fig. 4

Fig. 3. Scheme for the calculation of temperatures by the method of reflected heat sources.

Fig. 4. Deformation of wire at joint Δ, %, as a function of the distance to the crystal edge $l$, m•$10^{-5}$. 1) Contact welding by means of "split" electrode; 2) ultrasound welding; 3) dimensionless temperature θ as a function of dimensionless distance from crystal edge $l/r_0$.

which takes into account the reflection of heat from the plane y = 0 (Fig. 3). Thus, a semibounded body will be characterized by a system consisting of two parts divided by the plane y = 0, in each of which the heat distribution will be the same as in a body bounded by two adiabatic planes with one circular source.

The temperature in this system is given by

$$T(x, y, z) = T_1(x, y-l, z) + T_2(x, y+l, z) =$$
$$= \frac{q_0 r_0}{\lambda} \int_0^\infty J_1(x_0 r_0) \left[J_0\left(x_0 \sqrt{x^2+(y-l)^2}\right) + J_0\left(x_0 \sqrt{x^2+(y+l)^2}\right)\right] \exp(-x_0 z) \frac{dx_0}{x_0}.$$

In practice, it is important also to determine the temperature in the angular zone of the crystal where the additional hypothetical heat source acts. In the system of coordinates X, Y', Z' (Fig. 3)

$$T(x, y', z') = \frac{q_0 r_0}{\lambda} \int_0^\infty J_1(x_0 r_0) \left[J_0\left(x_0 \sqrt{(x-k)^2+(y'-l)^2}\right) + J_0\left(x_0 \sqrt{(x-k)^2+(y'+l)^2}\right) + \right.$$
$$\left. + J_0\left(x_0 \sqrt{(x+k)^2+(y'-l)^2}\right) + J_0\left(x_0 \sqrt{(x+k)^2+(y'+l)^2}\right)\right] \exp(-x_0 z') \frac{dx_0}{x_0}.$$

The procedure adopted is qualitatively confirmed by the increase in the deformation of the joint as the crystal face is approaches, which is due to the increase in yield of the joined materials because of the rise in temperature at contact (Fig. 4).

It is known [7, 8] that deformation of the wire by more than 40% leads to a reduction in the strength and reliability of the joint, associated with defects in the arrangement of active and passive elements of the semiconductor structure and a low output of suitable devices. Therefore, in choosing the optimal welding conditions for each specific type of device with a definite configuration of the contact areas, it is necessary to adjust the parameters of the welding pulse and to make allowance for the constructional features of the assembly equipment. Heat conditions are more critical for contact welding using a "split" electrode than for ultrasound welding (Fig. 4).

In connection with this, calculations of the temperature field should use Eq. (8) when $l > r_\infty$, $k > r_\infty$, Eq. (26) for $l < r_\infty$ or $k < r_\infty$, and Eq. (27) for the angular zone of the crystal where $l < r_\infty$ and $k < r_\infty$.

In the case of more complex boundary conditions, when Eq. (17) is not satisfied for a larger number of crystal faces, the temperature field may be calculated, by analogy with Eqs.

(26) and (27), by introducing additional hypothetical sources to account for heat reflected from the corresponding boundary planes. In most cases, the IS semiconductor crystal has dimensions $(0.5 \times 0.5 + 4 \times 4) \cdot 10^{-6}$ m$^2$ and, therefore, for the calculation of temperature fields during pulse microwelding, calculational schemes with one and three hypothetical sources are adequate.

## NOTATION

T, temperature; $r_0$, radius of the heat source; h, crystal thickness; $\lambda$, thermal conductivity; x, y, z, Cartesian coordinates; $q_0$, specific heat flux; $a$, b, semiaxes of the ellipse; dS, elementary area of the ellipse; r, radial variable; $\alpha$, heat-transfer coefficient; $\theta$, dimensionless temperature; $\xi$, $\chi$, dimensionless coordinates; Bi, Biot number; $\varepsilon$, emissivity; $\sigma$, Stefan–Boltzmann constant; $T_0$, initial temperature of the medium; k, $l$, distances from the center of the heat source to the crystal boundary planes.

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# SIMULATION OF THE THERMAL STATE OF A BILLET FROM CASTING TO ROLLING


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A mathematical model is given for the thermal treatment of a billet for rolling, in which the crystallization, cooling, and reheating are considered as stages in a single process.


Very complex effects occur in the heating and cooling of a billet during preparation for rolling; the time during which such a billet is suitable for rolling is governed by the size, type of mold, grade of steel, time spent in the mold, cooling time after removal from the mold, temperature in the heating oven, and so on. Many of these parameters may vary within wide limits. The product quality and the throughput are dependent on the parameter values.

Methods are available for calculating the individual stages in the process, and one can use these to calculate the crystallization, cooling, and heating with reasonable precision, but only if one has available reasonably reliable data on the initial state for each of the stages.

However, it has been shown [1-4] that division of the preparation into stages cannot provide all the necessary information, and it would be best to perform the calculations on


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